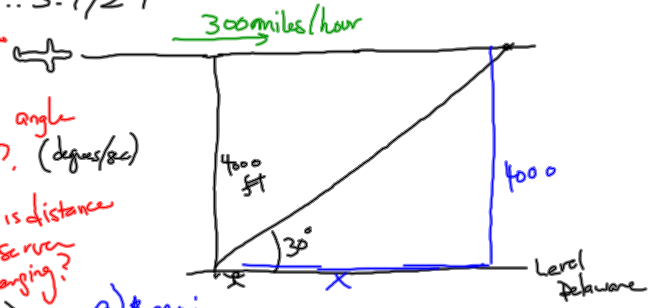
sidetrack 3.7/24

airplane



a) how fast is angle changing? (degrees/sec)

b) how fast is distance from observer changing? (ft/sec)

a) * eqn:

$$\tan\theta = \frac{4000 \text{ ft}}{x} = 4000x^{-1}$$

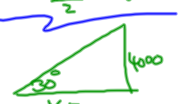
☆ $$\sec^2\theta \cdot \frac{d\theta}{dt} = 4000\left(-x^{-2} \cdot \frac{dx}{dt}\right)$$

$$\sec^2\theta \, \frac{d\theta}{dt} = \frac{-4000}{x^2} \, \frac{dx}{dt}$$

$\frac{dx}{dt} = 300$ mph

$\theta = 30°$

$\sec 30° = \frac{1}{\cos 30°} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$



$x =$

$\frac{4000}{x} = \tan 30° = \frac{1}{\sqrt{3}}$

$\Rightarrow x = 4000\sqrt{3}\left(\frac{1}{5280}\right) \Rightarrow$

☆ $$\frac{4}{3} \, \frac{d\theta}{dt} = \frac{-4000 \text{ ft}}{x^2}(300 \text{ mph})$$

$$\frac{d\theta}{dt} = \left(\frac{3}{4}\right)\left(-\frac{4000}{x^2}\right)\left(\frac{1}{5280}\right)(300)$$

$$\frac{d\theta}{dt} = \frac{3}{4}\left(\frac{-4000}{\left(\frac{4000\sqrt{3}}{5280}\right)^2}\right)\left(\frac{1}{5280}\right)(300)$$

$$\frac{d\theta}{dt} = -\frac{5280}{(4000)(3)}(300)\left(\frac{3}{4}\right) = -\frac{3}{160} \cdot 5280$$

$$\frac{d\theta}{dt} = -3.33 = 99 \text{ radians/hr}$$

$$= -99 \cdot \frac{180}{\pi} \cdot \frac{1}{3600}$$

$$= -\frac{99}{20\pi} \text{ degrees/second}$$

why radians?

$\frac{d}{dx}\sin x = \cos x$

$= \lim_{h\to 0} \frac{\sin(x+h) - \sin(x)}{h}$

relies on $\lim_{h\to 0} \frac{\sin h}{h} = 1$

3.7 again / 4b

$$x^2+y^2=2x$$

1 ✓

2 ✓

3 ⇒

$$\frac{dy}{dt}=3$$

$$(x,y)=\left(\frac{2+\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$$

$$2x\frac{dx}{dt}+2y\frac{dy}{dt}=2\frac{dx}{dt}$$

$$2\left(\frac{2+\sqrt{2}}{2}\right)\frac{dx}{dt}+2\left(\frac{\sqrt{2}}{2}\right)(3)=2\frac{dx}{dt}$$

$$(2+\sqrt{2})\frac{dx}{dt}+3\sqrt{2}=2\frac{dx}{dt}$$

$$\sqrt{2}\frac{dx}{dt}+3\sqrt{2}=0$$

$$\sqrt{2}\frac{dx}{dt}=-3\sqrt{2}$$

$$\therefore \frac{dx}{dt}=-3$$

4.2/31)

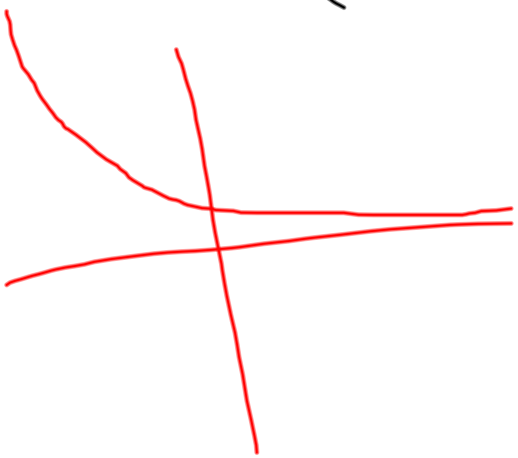
$$xe^{-x}+2e^{-x}=0$$

$x \cdot y = 10$

$x=10$

$y=10$

$$e^{-x}(x+2)=0$$



$$x+2=0$$

$$x=-2$$

1a $\ln\left(\frac{1}{x}\right) = -2$

$e^{\left[\ln\left(\frac{1}{x}\right)\right]} = e^{\left[-2\right]}$

$\frac{1}{x} = e^{-2} = \frac{1}{e^2} \quad \therefore x = e^2$

$\frac{1}{x} = \frac{e^{-2}}{1} \Rightarrow 1 = x \cdot e^{-2}$

$e^2 = 1 \cdot \frac{e^2}{1} = \frac{1}{\frac{1}{e^2}} \quad \cdots \quad \frac{1}{e^{-2}} = x$

21) $\log_5(5^{2x}) = 8$

$5^8 = 5^{2x}$

$\therefore 8 = 2x$

$4 = x$

$2x = 8$

$\log_a b = c$

$a^c = b$

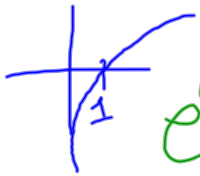
27) $5^{-2x} = 3$

$\ln(5^{-2x}) = \ln(3)$

$\dfrac{-2x \ln(5)}{-2\ln 5} = \dfrac{\ln(3)}{-2\ln 5}$

$x = \dfrac{\ln 3}{-2\ln 5} \approx -.341...$

25)



$$\ln\left(\frac{1}{x}\right)+\ln(2x^3)=\ln 3$$

$$e^{x+y}=e^x\cdot e^y$$

$$e^{\ln\left(\frac{1}{x}\right)}\cdot e^{\ln(2x^3)}=e^{\ln 3}$$

$$\left(\frac{1}{x}\right)\left(2x^3\right)=(3)$$

$$2x^2=3$$

$$x^2=\frac{3}{2}$$

$$x=\pm\sqrt{3/2}$$

But x can't be negative

so $x=+\sqrt{\frac{3}{2}}$

11b) $\ln\left(\frac{x^2\sin^3 x}{\sqrt{x^2+1}}\right)$

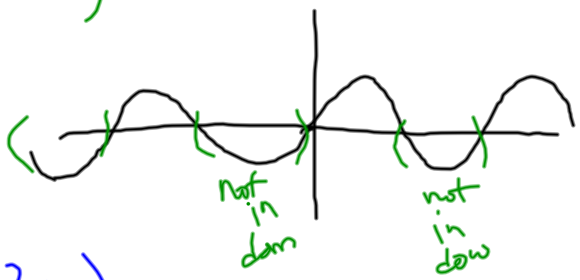
log rules:

$$\left.\begin{aligned}\ln(ab)&=\ln(a)+\ln(b)\\ \ln\left(\frac{a}{b}\right)&=\ln(a)-\ln(b)\\ \ln(a^b)&=b\ln(a)\end{aligned}\right\}$$

applies to $\log_b$ also

$$=\ln(x^2\sin^3 x)-\ln\left(\sqrt{x^2+1}\right)$$

$$=\ln(x^2)+\ln(\sin^3 x)-\ln(x^2+1)^{1/2}$$



$$=2\ln(x)+3\ln(\sin x)-\frac{1}{2}\ln(x^2+1)$$

but x^2 is always positive so I need $|x|$ (so I can always take ln)

31)

$xe^{-x} + 2e^{-x} = 0$

$\cancel{e^{-x}}(x + 2) = 0$

$x + 2 = 0$

$x = -2$

$xy = 10$
$x = 10$
or
$y = 10$

stoopyd

$\frac{xe^{-x}}{e^{-x}} = \frac{-2e^{-x}}{e^{-x}}$

$x = -2$

$x(e^{-x})$

$xe^{-x} = -2e^{-x}$

$-x^2 \ln e = 2x \ln e$

$-x^2 = 2x$

$-x = 2$

$x = -2$

$\log 8^2$

$2\log 8$

$\ln e^{-x}$

$-x \ln e$

$xe^{-x} = -2e^{-x}$

$\ln(xe^{-x}) \quad \ln(-2e^{-x})$

$\ln(x) + \ln(e^{-x}) \quad \ln(-2) + \ln(e^{-x})$

$= \ln(x) + -x\ln(e) \quad \ln(-2) - x$

$= \ln x - x$

2a

$$2e^{3x} = 7$$

$$\ln(e^{3x}) = \ln\left(\frac{7}{2}\right)$$

$$3x = \ln\left(\frac{7}{2}\right)$$

$$x = \frac{1}{3}\ln\left(\frac{7}{2}\right) = \frac{\ln\left(\frac{7}{2}\right)}{3}$$

19)

$$e^{\ln\left(\frac{1}{x}\right)} = e^{-2}$$

$$\frac{1}{x} = e^{-2} = \frac{1}{e^2}$$

$$x = e^2$$

$$\frac{1}{2^3} \neq \frac{1}{2^4}$$

$$\frac{1}{x} = e^{-2}$$

$$1 = (e^{-2})(x)$$

$$\frac{1}{e^{-2}} = x$$

$$\frac{e^2}{1} = x$$

4.2 homework

2010-11-03 Pd 3